Measurement of the Interlamellar Spacing of Pearlite

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Following a historical review of methods that have been employed for measuring the interlamellar spacing of pearlite, these methods are compared with reference to a continuously cooled AISI 1040 carbon steel specimen. Extensive analyses, using TEM thin foils and stereological analysis of TEM replicas, were conducted to determine the mean true spacing. Pearshall’s partial resolution method, employing light microscopy, agreed well with the analysis as long as the method was rigorously controlled. The TEM method of searching for the finest pearlite colony as an estimate of the true spacing produces variable results with no fixed relationship to the mean true spacing. The analytical method of Roösz et al. for developing the distribution of true spacings produced reasonable agreement with the stereological estimate of the mean true spacing.

Introduction

Measurements of the interlamellar spacing of pearlite in steels have been commonly employed in the development of structure-property correlations for pearlitic steels. The interlamellar spacing can be related to the heat treatment conditions and to mechanical properties such as the tensile strength, hardness, and toughness. Prior to the development of the electron microscope, measurements were limited by the resolution capability of the optical microscope or were performed only on lamellae nearly parallel to the plane of polish. In a few cases, where the interlamellar spacing was quite coarse, actual measurements were possible. Since the development of the transmission electron microscope, optical methods have been used infrequently. However, a number of procedures have been employed that produce different estimates of the mean true interlamellar spacing $d_t$. 
Interlamellar spacing measurements are complicated by the spacing variations within a given pearlite colony, those between different colonies, and those produced by the metallographic sectioning plane with respect to the orientation of the lamellae. When the sectioning plane is perpendicular to the lamellae, the true interlamellar spacing of the colony can be determined. As the sectioning plane becomes less perpendicular and approaches an orientation parallel to the lamellae, the apparent spacing and the apparent carbide thickness increase dramatically. Furthermore, pearlite formed during continuous cooling exhibits a wider range of true or apparent spacings than pearlite formed isothermally.

**Measurement Methods**

Belaiew [1, 2] was among the first to consider the problems associated with interlamellar spacing measurements in pearlite. He mistakenly concluded that the true interlamellar spacing within a sample was constant and that the observed variation was due solely to the sectioning plane effect. Belaiew [2] assumed that if the true interlamellar spacing were smaller than the resolution limit of the optical microscope, about 0.2 μm, then the resolvable portion of the lamellae could be related to the true spacing through the distribution of apparent spacings created by the effect of the sectioning plane orientation.

In samples in which the true spacing was less than the resolution limit of the light microscope, a few areas can be found where the pearlite lamellae are resolvable. The apparent spacing of these colonies is much larger than the true spacing. Belaiew noted that the edges of the carbides in these colonies appeared to be broken or frayed. Using a sample in which all of the pearlite was resolvable, Belaiew determined that this frayed condition obtained when the sectioning plane was 83° from the plane perpendicular to the lamellae. Hence, to determine the interlamellar spacing of fine pearlite, Belaiew measured the apparent spacing of the frayed carbide colonies, assumed that they were at an angle of 83° from the perpendicular, and then calculated the interlamellar spacing by dividing the apparent spacing of the frayed carbides by the secant of 83° (8.2055).

Greene [3] optically examined pearlite in rail steel at 2500–5000× and measured the spacing of the finest colony, which he assumed to be the true interlamellar spacing. Again the assumption was made that colony spacing was constant, with all of the observed variation due to the sectioning plane.

Pellissier et al. [4] showed that the interlamellar spacing of pearlite, even in isothermally transformed pearlite, is not constant but rather ex-
Pearlite Interlamellar Spacing Measurement

hibits a statistical distribution of spacings about a mean interlamellar spacing \( \bar{\sigma}_0 \). This analysis of isothermally transformed pearlite revealed that the mean spacing \( \bar{\sigma}_0 \) was 1.65 times the minimum observed spacing. However, this value does not appear to be a universal constant. In studies of pearlite in 0.1%-0.2% carbon steels, Birkbeck and Wells [5] found the ratio of the mean spacing to the finest observed spacing to vary between 1.1 and 1.8. Asundi and West [6] observed a mean spacing of 2-2.5 times the minimum spacing.

Pearsall [4] developed a procedure, referred to as the partial resolution method, for estimating \( \bar{\sigma}_0 \) using the resolution of the light microscope. With an objective of known resolution \( d \), the area fraction of unresolvable pearlite was measured by point counting. Next the mean spacing \( \bar{\sigma}_0 \) was calculated by

\[
\bar{\sigma}_0^2 = d^2(1 - V_V^2),
\]

where \( V_V \) is the volume fraction of pearlite with spacing finer than \( d \); \( d = 0.5\lambda/NA \), where \( \lambda \) is the wavelength of light; and \( NA \) is the numerical aperture of the objective. Gregory et al. [7] evaluated Pearsall’s partial resolution method and achieved best results when the volume fraction of unresolved pearlite was 0.5-0.65.

The method of Brown and Ridley [8], employing TEM replicas, has been widely used to determine the minimum interlamellar spacing \( \sigma_{\text{min}} \) or an average minimum spacing \( \bar{\sigma}_{\text{min}} \). The replica is scanned until the finest pearlite colony is found. The magnification is adjusted so that this colony is resolvable. Then, the number \( n \) of lamellae crossed at right angles to the diameter \( d_c \) of a circle inscribed on the viewing screen is counted, and the minimum spacing \( \sigma_{\text{min}} \) is calculated by

\[
\sigma_{\text{min}} = (1/n)(d_c/M).
\]

The magnification \( M \) is determined using a replica of a diffraction grating of known spacing. The average minimum spacing is the mean of a number of such measurements.

Although the minimum spacing is related to the mean true spacing, the relationship is not constant. Also, the spacing of the finest observed colony varies with the effort expended in the search. Hence, these two variables may reduce the ability to compare results obtained by different experimenters and decrease reproducibility. Nevertheless, some workers [9] state that the minimum spacing procedure is the most reliable and consistent method of determining the interlamellar spacing.

Underwood [10] recommends determining the mean random spacing \( \bar{\sigma}_r \) to estimate the mean true spacing \( \bar{\sigma}_t \). Test lines are applied at random to the microstructure. A circular test grid of diameter \( d_c \) is ideal for this
The number $n$ of intersections of carbides with test lines is counted. This procedure is repeated on a number of fields chosen without bias. The number of intersections per unit length of test line, $N_L$, is calculated by dividing $n$ by the true total test line length per measurement field. Then, the mean random spacing $\bar{a}_r$ is calculated from

$$\bar{a}_r = \frac{1}{N_L} = \frac{\pi d_c/M}{n}.$$  \hspace{1cm} (3)

The circular test grid is ideal for the determination of $\bar{a}_r$ as it guarantees orientation randomization and provides a constant test line length for each field measurement. These two criteria are much more difficult to satisfy when using a grid with straight test lines. Error and bias can be introduced if the test line length per measurement varies significantly.

When the volume fraction of pearlite is below 0.5, the magnification must be increased so that most of the pearlite grains are larger than the apparent test grid area. The test grid should not overlap the pearlite and cover proeutectoid ferrite.

As the magnification is raised and the average number $n$ of intersections per field decreases, measurement accuracy decreases. In general, best accuracy is obtained when the lowest feasible magnification is used. Such a magnification, however, must be high enough for adequate resolution and accurate counting but should be no higher than necessary so that $n$ may be maximized.

To obtain a good estimate of $\bar{a}_r$, a number of measurements must be made using unbiased field selection. This can be accomplished by moving the $x$ and $y$ stage controls without looking at the image. The fields should be spaced in a systematic pattern around the sample surface. One should not deliberately attempt to select fields that appear to be typical, minimum, or maximum in spacing. Measurement accuracy improves as the number of measurements increases.

Saltykov [11] has shown that for pearlite with a constant spacing within each colony the mean true spacing $\bar{a}_t$ is related to the mean random spacing $\bar{a}_r$ by

$$\bar{a}_t = 0.5\bar{a}_r.$$  \hspace{1cm} (4)

Although this relationship is not universally exact, it is an excellent approximation. Gensamer et al. [12] found the ratio of $\bar{a}_r/\bar{a}_t$ to vary between 1.9 and 2.0.

Recently an analytical procedure has been described [13–15] for the determination of the distribution of true interlamellar spacings; this procedure also permits calculation of the mean true spacing. This calculation is more time consuming than the stereological approach of Underwood
and Saltykov; however, the method does produce the entire distribution of true spacings, rather than just the mean value. Cahn and Fullman [16] have also published an analytical procedure for obtaining the distribution of spacings.

**Application of the Methods**

To illustrate the use of these methods, an as-rolled sample of AISI 1040 carbon steel was evaluated to determine the interlamellar spacing of the pearlite constituent. Figure 1 shows an optical micrograph of the sample and illustrates the fineness of the pearlite, the sectioning plane influence, and the spacing variation within a given colony.

The two optical procedures we have described were used to determine the interlamellar spacing. Using Belaiw's secant method the sample was scanned at 1000× and a dozen measurements of frayed, highly oblique colonies were made. The average spacing of the frayed lamellae was $2300 \pm 690$ nm (with 95% confidence limit) with a range from 1000 to 3300 nm.

![Optical micrograph of the ferrite-pearlite structure in continuously cooled AISI 1040 steel (4% picral etchant).](image)
This average was divided by the secant of 83°, producing 280 nm as an estimate of the interlamellar spacing. The relative accuracy of this measurement was about 30%.

Pearsall's partial resolution method was tried first using a 100× dry objective with a 0.95 numerical aperture. The finest resolvable detail with this objective would have a calculated minimum spacing of 290 nm. Ten fields at 1000× were point counted using a 100-point grid to determine the amount of unresolvable pearlite (50.7%). Equation (1) was used to calculate a mean spacing of $\bar{\sigma}_0 = 250$ nm. A second attempt, using a 50× dry objective with a 0.85 numerical aperture, produced an estimate of $\bar{\sigma}_0 = 195$ nm. This latter value, as will be shown, is not a good estimate, and we must agree with Gregory et al. [7] that the amount of unresolved pearlite present significantly influences test accuracy.

Several procedures using the TEM were employed to estimate the interlamellar spacing. Replicas of the etched sample (Fig. 2) were examined by two different operators. The finest colonies measured had spacings of 175 and 168 nm for the two operators. If it is assumed that the finest spacing multiplied by 1.65 produces an estimate of the mean true spacing, estimates of 289 and 277 nm, respectively, are obtained.

Next, a large number of measurements were made on pearlite colonies chosen systematically but without bias. Measurements were made using test lines oriented perpendicularly to the lamellae ("directed measurements"). The number of carbide lamellae was counted over a fixed dis-

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**Fig. 2.** Transmission electron microscope replica of pearlite in AISI 1040 steel.
Pearlite Interlamellar Spacing Measurement

The true test line length was divided by \( n \) to determine the apparent spacing in each colony. Three such measurements were made in each colony. The average of 183 such measurements was 333 nm. The finest colony spacing was 124 nm, considerably finer than the result of the initial examination. The 183 measurements with the oriented test lines were grouped into 12 classes to prepare a cumulative frequency plot (Fig. 3).

The same colonies that were measured using the directed test lines were also measured using a circular test line to estimate the mean random spacing \( \bar{\sigma}_r \). The average of 92 such measurements was 508 nm. Then, according to equation (4), the mean true spacing \( \bar{\sigma}_t = 254 \) nm, which agrees closely with the estimate using Pearsall's partial resolution method. The random spacing measurements were grouped into 15 classes and plotted as a cumulative frequency curve (Fig. 3). Both distributions in Fig. 3 are nonnormal. The average of the ratios of the random spacing to the directed spacing for the same colony was 1.53, approximately \( \pi/2 \). This agrees with stereological principles (see the Appendix).

The sample was deeply etched with picral and examined in the scanning electron microscope (Fig. 4). First, 15 micrographs were made of colonies that appeared to be approximately perpendicular to the surface. The sample was not tilted but was normal to the electron beam. The average of
directed spacing measurements on these micrographs was 250 nm, which agrees with the stereological random measurements (TEM replicas) used to estimate $\bar{d}_r$. Next, 15 additional micrographs were made using systematic unbiased field selection. The circular test grid was used to measure the mean random spacing, 510 nm, producing an estimate of 255 nm for the mean true spacing. This value also agrees well with the more exhaustive TEM replication estimate of $\bar{d}_r$. The average directed spacing of the pearlite on the randomly selected fields was 320 nm, rather close to the value of 333 nm obtained on the randomly chosen TEM replicas. The ratio of random to directed measurements for the 15 random SEM fields was 1.53, again quite close to $\pi/2$.

Next, thin foils (Fig. 5) were prepared from an area adjacent to that replicated and examined in the TEM. Colonies in which the lamellae appeared to be perpendicular to the foil surface were photographed. Straight oriented test lines were used to measure the interlamellar spacings at three locations within each colony. The average of 218 such measurements was $254.5 \pm 9.0$ nm (with 95% confidence limit), which agrees very closely with the stereological estimate of $\bar{d}_r$ from $\bar{d}_r$ on replicas. The finest observed spacing was 100 nm, again much finer than the spacing detected in the initial search on the replicas. The directed measurements
Fig. 5. Transmission electron micrograph of thin foil of pearlite in AISI 1040 steel.

Fig. 6. Cumulative frequency distribution of directed interlamellar spacing measurements of perpendicular lamellae made on foils. [For σ₁ one has a mean of 214.3 nm ± 9.9 nm (with 95% confidence limits), a minimum of 100 nm, and a maximum of 450 nm, based on n = 218.]
of the true spacings made on the foils were grouped into ten classes to prepare a cumulative frequency distribution curve (Fig. 6). A normal data dispersion is apparent, with data ranging from 100 to 450 nm.

The directed and random spacing measurements using TEM replicas and the true spacing measurements using thin foils were further analyzed to assess the nature of the distributions of the three types of measurements. Each group of data was arranged in ascending spacing order and converted to cumulative percentages. The cumulative percentages as a function of interlamellar spacing were plotted on probability paper (Fig. 7). The true spacing measurements on foils exhibited a normal data distribution, while the random and directed measurements made on replica exhibited log-normal data distributions.

In the extensive TEM replication and foil studies, the finest observed interlamellar spacing was 124 nm for replicas and 100 nm for foils. Comparing these results with those of the initial search, we must conclude that as the degree of effort in searching is increased, the spacing of the finest observed lamellae decreases. Hence, this procedure is suspect.

A reliable reproducible method for estimating interlamellar spacing, as there is no universal method to convert such information to estimate the actual mean true spacing. For the sample studied (continuously cooled A1 1040), the ratio of the mean true spacing, 254.5 nm, to the finest observed spacing was in the range of 2.0–2.5 observed by Asundi and West [6].

For isothermally transformed pearlite this ratio is undoubtedly lower. As a final step, the method of Bian [13] and Roosz and Gács [14, 1] was used to analyze further the 138 directed measurements made on such hematic unbiased fields (TEM replicas). This method is complicated, and so the calculation procedure will now be described.

The minimum and maximum observed spacings, \( \sigma_{\text{min}} \) and \( \sigma_{\text{max}} \), were 124 and 1130 nm, respectively. Hence, if we decide to group the data into ten classes, the difference in each class is

\[
\Delta x = (1130 - 124)/10 = 100.6 \text{ nm.}
\]
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As a final step, the method of Baan [13] and Roós and Gácsi [14, 15] was used to analyze further the 183 directed measurements made on systematic unbiased fields (TEM replicas). This method is complicated, and so the calculation procedure will now be described.

The minimum and maximum observed spacings, $\sigma_{\text{min}}$ and $\sigma_{\text{max}}$, were 124 and 1130 nm, respectively. Hence, if we decide to group the data into ten classes, the difference in each class is

$$\Delta \sigma = (1130 - 124)/10 = 100.6 \text{ nm},$$
TABLE 1
Calculation of the Distribution of True Spacings

<table>
<thead>
<tr>
<th>Class</th>
<th>Limits (nm)</th>
<th>Number</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
<th>$A_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$</td>
<td>100.6 - 201.3</td>
<td>23</td>
<td>0.1257</td>
<td>0.1257</td>
<td>0.145</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>201.3 - 301.95</td>
<td>80</td>
<td>0.4372</td>
<td>0.5629</td>
<td>0.5716</td>
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<tr>
<td>$\sigma_3$</td>
<td>301.95- 402.6</td>
<td>45</td>
<td>0.2459</td>
<td>0.8088</td>
<td>0.2671</td>
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<tr>
<td>$\sigma_4$</td>
<td>402.6 - 503.25</td>
<td>14</td>
<td>0.0765</td>
<td>0.8853</td>
<td>0.0161</td>
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<tr>
<td>$\sigma_5$</td>
<td>503.25- 603.9</td>
<td>9</td>
<td>0.0492</td>
<td>0.9345</td>
<td>0.0247</td>
</tr>
<tr>
<td>$\sigma_6$</td>
<td>603.9 - 704.55</td>
<td>2</td>
<td>0.0109</td>
<td>0.9454</td>
<td>-0.0255</td>
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<td>$\sigma_7$</td>
<td>704.55- 805.2</td>
<td>5</td>
<td>0.0273</td>
<td>0.9727</td>
<td>0.0334</td>
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<td>$\sigma_8$</td>
<td>805.2 - 905.85</td>
<td>3</td>
<td>0.0164</td>
<td>0.9891</td>
<td>0.0081</td>
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<td>$\sigma_9$</td>
<td>905.85-1006.5</td>
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<td>0.005</td>
<td>0.9941</td>
<td>-0.0107</td>
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<tr>
<td>$\sigma_{10}$</td>
<td>1006.5 -1107.15</td>
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<td>0</td>
<td>0.9941</td>
<td>0</td>
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<tr>
<td>$\sigma_{11}$</td>
<td>1107.15-1207.8</td>
<td>1</td>
<td>0.005</td>
<td>0.9991</td>
<td>0.0008</td>
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<td></td>
<td>183</td>
<td></td>
<td></td>
<td></td>
<td>1.0306</td>
</tr>
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</table>

where

$$\Delta \sigma = \sigma_i - \sigma_{i-1} = \sigma_1/a.$$ 

The value of $a$ must satisfy the relationship

$$\sigma_1 = a \Delta \sigma < \sigma_{\text{min}}.$$ 

To determine $a$, one divides $\sigma_{\text{min}}$ by $\Delta \sigma$:

$$a' = \frac{124}{100.6} = 1.23.$$ 

The value of $a$ should be a positive integer less than $a'$; hence, $a = 1$ in this calculation.

Next, size classes are established to classify the 183 measurements into groups. Class 1, $\sigma_1$, covers measurements from 100.6 to 201.3 nm; class 2, $\sigma_2$, covers 201.3-301.95 nm; and so forth, as shown in Table 1. Column 3 of Table 1 lists the number of measurements per class; column 4 lists the frequency of measurements in each class, and column 5 is the cumulative frequency.

Next, along with Table 1 of Roósz and Gácsi [15], the inverse coefficient matrices for $a = 1$ allow one to calculate the frequency distribution of the true interlamellar spacing per class, $A_i$. For example, $A_1$ is calculated from

$$A_1 = (1.1547)(0.1257) = 0.145.$$
PEARLITE INTERLAMELLAR SPACING MEASUREMENT

Fig. 8. Frequency function of the true interlamellar spacings of lamellae measured using the method of Roosz et al. (a = 1, σ = 236 nm.)
Similarly

\[ A_2 = (-1.4606)(0.1257) + (1.3416)(0.5629) \]
\[ = 0.5716, \]

\[ A_3 = (0.2221)(0.1257) + (-1.7566)(0.5629) + (1.5119)(0.8088) \]
\[ = 0.2671. \]

This process is continued to generate the values shown in column 6 of Table 1. This analytical method produces some negative values of \( A_i \) in the high-class numbers, and the sum of the \( A_i \) values is generally greater than 1. These two problems arise from the limited data; i.e., about 1000

<table>
<thead>
<tr>
<th>Measurement method</th>
<th>Measured (or calculated) values</th>
<th>( \sigma_{\text{min}} )</th>
<th>( \tilde{\sigma}_d )</th>
<th>( \tilde{\sigma}_r )</th>
<th>( \sigma_{\text{min}} )</th>
<th>( 2\sigma_{\text{min}} )</th>
<th>( 2.5\sigma_{\text{min}} )</th>
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<td>(336)</td>
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<td>508(^a)</td>
<td>(254)(^b)</td>
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<tr>
<td>directed test grid</td>
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<td>(523)(^c)</td>
<td>(262)(^d)</td>
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<tr>
<td>finest measured spacing method of Baan et al.</td>
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<td>(248)</td>
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<td>colonies perpendicular to surface</td>
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<td></td>
<td></td>
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<td>254.5(^a)</td>
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<td>250</td>
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\(^a\) Based on the best measurements of the mean true spacing and the mean random spacing, the ratio of \( \tilde{\sigma}_t/\tilde{\sigma}_1 = 508/254.5 = 1.996 \), i.e., essentially identical to the value predicted by Saltykov [11] and verified by Gensamer et al. [12].

\(^b\) Calculated from mean random spacing by (4).

\(^c\) Calculated from mean directed spacing by (A4).

\(^d\) Calculated from mean directed spacing by (A5).
measurements are required to develop a highly accurate distribution of true spacings. Figure 8 shows the frequency function curve of the true spacings.

The final step is to calculate the average true interlamellar spacing, which is the sum of the products of $A_i$ values by $\sigma_i$ values. For this example, we calculated $\bar{\sigma}$ as 236 nm, which is slightly lower than the stereological estimate. The test data are summarized in Table 2.

**Discussion**

Analysis of the interlamellar spacing of pearlite in continuously cooled AISI 1040 carbon steel by different published methods has revealed difficulties with some of these methods. The optical method of Belaiiew overestimated the true spacing. This may be a result of the assumption that the spacing of all colonies is constant, with all of the observed apparent spacing differences being due to the sectioning plane effect. Pearsall's partial resolution method produced a good estimate of the mean true spacing when the amount of unresolved pearlite was in the range 50%–65%. With a higher amount of unresolved pearlite, the estimate was poor. This places a severe restriction on the analysis procedure. Additionally, the point count of the amount of unresolved pearlite is highly subjective and depends on factors other than merely the numerical aperture of the objective and the wavelength of the light.

Electron metallographic procedures are much better suited to measuring the interlamellar spacing since the resolution is high enough to ensure complete resolution and accurate counting. The method of searching for the finest pearlite colony as a measure of the spacing is clearly dependent on the amount of effort extended. The greater the effort, the finer the observed spacing. Conversion of the finest spacing to a mean true spacing requires the use of a multiplying factor that is not a constant. This procedure must be seen as susceptible to error and lack of reproducibility.

The simplest, most universal procedure to determine the interlamellar spacing is to measure the mean random spacing using unbiased systematic field selection and randomly oriented test lines, e.g., a circular test grid. The mean true spacing is simply one-half the mean random spacing. To use this method, unbiased field selection and unbiased grid placement must be employed. The magnification must be high enough to resolve the lamellae for accurate counting but no higher than necessary to maximize the $n$ counts per field. A constant grid line length per field measurement prevents bias. This stereological method is highly reproducible, simple, and efficient.
The measurements have shown that there is a fixed relationship between random and directed (perpendicular) spacing measurements made on the same fields. The directed measurement multiplied by $\pi/2$ is approximately equal to the random measurement. This observation could be used to convert directed measurements to random measurements and then to a mean true spacing. Since the mean random spacing is twice the mean true spacing, it follows that the mean directed spacing multiplied by $\pi/4$ gives the mean true spacing.

The method of Baan, Roósz, and Gáczi for calculating the distribution of true spacings permits the development of a frequency plot of the true spacings and an estimate of the mean true spacing. If the mean true spacing alone is desired, the stereological method is faster and more accurate. In studies of the growth of pearlite, however, it may be useful to be able to obtain the entire distribution curve of the true spacings.

Appendix: The Relationship Between $\bar{\sigma}_r$ and $\bar{\sigma}_d$

If a single pearlite colony is cut by a test circle of diameter $d_c$ (Fig. 9), then the mean directed spacing $\bar{\sigma}_d$ is

$$\bar{\sigma}_d = d_c/n_d,$$

where $n_d$ is the number of carbides intercepted by the diameter $d_c$. In Figure 9 $n_d = 9$. The mean random spacing $\bar{\sigma}_r$ is given by

$$\bar{\sigma}_r = \pi d_c/n_r,$$

where $n_r$ is the number of carbides intercepted by a random line of length $\pi d_c$. In Figure 9 $n_r = 18$. In general, $n_r \sim 2n_d$; in this example they are

![Fig. 9. Schematic illustrating a pearlite colony intersected by a circular test grid of diameter $d_c$.](image)
Pearlite Interlamellar Spacing Measurement

equal. Hence,

\[ \tilde{\sigma}_r = \pi d_c / 2n_d. \]  

(A3)

Since \( d_c = n_d \tilde{\sigma}_d \),

\[ \tilde{\sigma}_r = (\pi/2) \tilde{\sigma}_d; \]  

(A4)

that is, the ratio \( \tilde{\sigma}_r / \tilde{\sigma}_d = \pi/2 \).

Similarly, since the mean random spacing \( \tilde{\sigma}_r \) is twice the mean true spacing \( \tilde{\sigma}_t \), and the mean random spacing \( \tilde{\sigma}_r = (\pi/2) \tilde{\sigma}_d \), then

\[ \tilde{\sigma}_t = (\pi/4) \tilde{\sigma}_d. \]  

(A5)

References


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